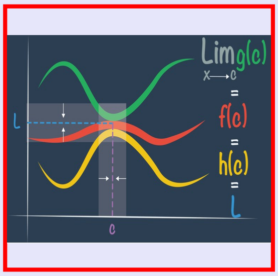
Calculus I

Lecture 31



Feb 19-8:47 AM

Use quadratic approximation to evaluate $\sin 32°$.

$$\boxed{f(x) \approx f(a) + f'(a)(x-a)} + \frac{f''(a)}{2}(x-a)^2$$

Linear Approximation

$\sin 32° \approx \sin 30° = \frac{1}{2}$

Let $f(x) = \sin x$

$a = 30° = \frac{\pi}{6}$

$$\sin x \approx \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2$$

$$\sin 32° \approx \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot 2° - \frac{1}{4} \cdot (2°)^2$$

convert to radians

$f(30°) = \sin 30° = \frac{1}{2}$

$f'(x) = \cos x \qquad f'(30°) = \cos 30° = \frac{\sqrt{3}}{2}$

$f''(x) = -\sin x \qquad f''(30°) = -\sin 30° = \frac{-1}{2}$

$180° = \pi$ Rad

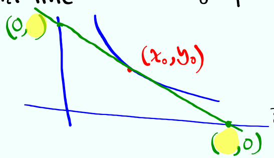
$1° = \frac{\pi}{180}$ Rad $\qquad 2° = \frac{\pi}{90}$ Rad.

$$\sin 32° \approx \boxed{\frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\pi}{90} - \frac{1}{4} \cdot \left(\frac{\pi}{90}\right)^2} \approx \boxed{.5299}25372$$

$\rightarrow \boxed{.5299}192642$

Oct 22-7:26 AM

Show that the Sum of x-Int ⁝ y-int of any tan. line to the graph of $\underbrace{\sqrt{x}+\sqrt{y}=\sqrt{C}}$ is C.



$x^{1/2}+y^{1/2}=c^{1/2}$

$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\frac{dy}{dx}=0$

$y^{-1/2}\frac{dy}{dx}=-x^{-1/2}$ $\qquad \frac{dy}{dx}=-\frac{x^{-1/2}}{y^{-1/2}}$ $\qquad \frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$

at any Point (x_0,y_0)

$m_{\text{tan. line}}=\frac{dy}{dx}\Big|_{(x_0,y_0)}=-\frac{\sqrt{y_0}}{\sqrt{x_0}}$

$y-y_0=-\frac{\sqrt{y_0}}{\sqrt{x_0}}(x-x_0)$ Eqn. of tan. line.

x-Int $\quad y=0 \quad -y_0=-\frac{\sqrt{y_0}}{\sqrt{x_0}}(x-x_0)$

$-y_0\sqrt{x_0}=-\sqrt{y_0}\,x+x_0\sqrt{y_0}$

$x=\frac{x_0\sqrt{y_0}+y_0\sqrt{x_0}}{\sqrt{y_0}}$

$x=\frac{x_0\sqrt{y_0}}{\sqrt{y_0}}+\frac{y_0\sqrt{x_0}}{\sqrt{y_0}}$

$x=x_0+\sqrt{x_0y_0}$ ← x-Int.

Oct 22-7:37 AM

Y-Int $\quad x=0 \quad y-y_0=-\frac{\sqrt{y_0}}{\sqrt{x_0}}(0-x_0)$

$y-y_0=\sqrt{y_0}\sqrt{x_0}$

$y=\sqrt{y_0}\sqrt{x_0}+y_0$ Y-Int

show sum of x-Int ⁝ y-Int = C

$x_0+\sqrt{x_0y_0}+\sqrt{x_0y_0}+y_0=$

$x_0+2\sqrt{x_0}\sqrt{y_0}+y_0=$

$(\sqrt{x_0})^2+2\sqrt{x_0}\sqrt{y_0}+(\sqrt{y_0})^2=\left[\sqrt{x_0}+\sqrt{y_0}\right]^2$

$=\left[\sqrt{C}\right]^2=\boxed{C}$

Oct 22-7:47 AM

Find $\frac{dy}{dx}\Big|_{(1,1)}$ if $x^3 + y^3 = 2xy$

$$3x^2 + 3y^2\frac{dy}{dx} = 2\left[1\cdot y + x\cdot\frac{dy}{dx}\right]$$

$$3y^2\frac{dy}{dx} - 2x\frac{dy}{dx} = 2y - 3x^2$$

$$\frac{dy}{dx} = \frac{2y-3x^2}{3y^2-2x}$$

$$\frac{dy}{dx}\Big|_{(1,1)} = \frac{-1}{1} = \boxed{-1}$$

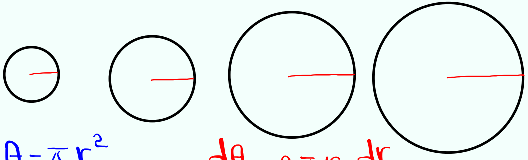
Oct 22-7:54 AM

Area of a Circle is increasing at the rate of $\frac{10}{\pi}$ cm²/min.

$\frac{dA}{dt} = \frac{10}{\pi}$ cm²/min

How fast is the circumference increasing when radius is 5cm?

$\frac{dC}{dt} = ?$ when $r = 5$ cm.



$A = \pi r^2$

$C = \pi\cdot 2r$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$

$\frac{dC}{dt} = 2\pi\cdot\frac{1}{\pi^2} = \boxed{\frac{2}{\pi}\text{ cm/min}}$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\overset{\checkmark}{\frac{10}{\pi}} = \overset{\checkmark}{2\pi}\cdot\overset{\checkmark}{5}\frac{dr}{dt}$

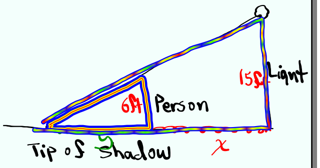
$\frac{dr}{dt} = \frac{1}{\pi^2}$ cm/min

Oct 22-8:00 AM

A street light is 15-ft tall.

A person 6-ft tall walks away from the light at speed of 5 ft/sec. along a Straight Path.

How fast is the tip of person's shadow changing when the person is 40 ft from the light.



$$\frac{y}{2\cancel{6}} = \frac{x+y}{\cancel{15}\,5}$$

$$5y = 2x + 2y$$

$$3y = 2x$$

Now take derivative with respect to time.

finish it, and we go over Your answer tomorrow.

Oct 22-8:07 AM

$$f(x) = \frac{x^3}{x^2+1}$$

1) Domain $(-\infty,\infty)$ No V.A.

2) x-Int & Y-Int $(0,0)$

3) $f(-x) = \frac{(-x)^3}{(-x)^2+1} = \frac{-x^3}{x^2+1} = -\frac{x^3}{x^2+1} = -f(x)$

$f(-x) = -f(x)$

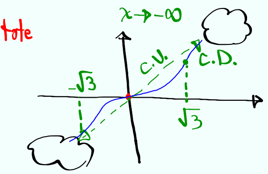
odd function → symmetric w/t origin

$$f(x) = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}$$

$$\lim_{x\to\infty} \frac{x}{x^2+1} = 0$$

$$\lim_{x\to\infty} \Big[\underset{\uparrow}{x} - \overset{\to 0}{\frac{x}{x^2+1}}\Big] = \infty \qquad \lim_{x\to-\infty} \Big[x - \overset{\to 0}{\frac{x}{x^2+1}}\Big] = -\infty$$

Slant Asymptote



$$f(x) = x - \frac{x}{x^2+1}$$

$$f'(x) = \frac{x^2(x^2+3)}{(x^2+1)^2} \geq 0$$

$f'(x) = 0 \rightarrow x^2 = 0 \rightarrow x = 0$

$$f''(x) = \frac{2x(3-x^2)}{(x^2+1)^3}$$

$f''(x) = 0 \rightarrow x = 0,\ x = \sqrt{3},\ x = -\sqrt{3}$

Oct 22-8:16 AM

Use ε & δ to prove $\lim_{x\to 1}(x^3+x^2)=2$ ✓

$f(x)=x^3+x^2$

$\lim_{x\to 1}(x^3+x^2)=1^3+1^2=2$

$a=1$

$L=2$

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^3+x^2-2|<\varepsilon$ whenever $|x-1|<\delta$

$\left|\underset{\text{Bound}}{(x^2+2x+2)}\underset{\text{Keep}}{(x-1)}\right|<\varepsilon$, $|x-1|<\delta$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & 0 & -2 \\ & & 1 & 2 & 2 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

If $\delta\le 1$

$-1<x-1<1$

$0<x<2$

if $|x^2+2x+2|=c$, $|x-1|<\frac{\varepsilon}{c}$

$|x^2+2x+2|<2^2+2(2)+2=10$

$\delta=\min\left\{1,\frac{\varepsilon}{10}\right\}$

Oct 22-8:30 AM